Strategies for Teaching Arithmetic: What Are the Facts?

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Students with a specific learning disability are a heterogeneous group with a wide variety of disorders. However, the individuals within this diverse group typically demonstrate a specific academic deficit. While the majority of research targeting this student population has focused on deficits in reading performance, investigations of general academic subtypes have consistently isolated arithmetic disabilities (Silver, Pennett, Black, Fair, & Balise, 1999). Students with isolated arithmetic disabilities display a unique pattern of verbal strengths, visual-perceptual-organizational weaknesses, and attention deficits. It has been speculated that inattention affects the acquisition of arithmetic skills (Marshall, Schafer, O'Donnell, Elliott, & Handwerk, 1999), specifically the memorization of basic number facts (Ackerman, Anhalt, & Dykman, 1986) ultimately compromising the acquisition of higher math skills (Gagne, 1983). Other factors contributing to an overall low performance in mathematics for these students include inappropriate instruction or conceptualization difficulties (Cawley, 1979).

Without mastery of basic facts, students often rely on an inefficient counting procedure such as using one's fingers. According to Mercer and Miller (1992), the benefits of automaticity or fluency in basic facts include "improved retention, ability to compute and/or solve higher level problems, finishing timed tests, completing homework faster, receiving higher grades, and developing positive feelings about math" (p.22). Since all symbolic algorithms require automatic retrieval of number facts, students with a learning disability become less proficient at learning new math algorithms associated with problem solving.

Contributing to this difficulty are instructional approaches and curriculum materials, which fail to: a) integrate basic facts with problem solving, and b) involve manipulatives or concrete experiences for promoting the development of basic computational skills. Thus, students may be limited in developing an awareness of number patterns that could aid them in becoming fluent with arithmetic facts.

This article offers a strategy for organizing arithmetic multiplication facts into a format that can more easily be learned by students with a specific learning disability. The emphasis on rules and number patterns encourages logical thinking, while the reduced emphasis on drill and memorization techniques prevents boredom. Foremost to any instruction in mathematics, however, should be the introduction of "real life" math problems in which learning number facts provides practical solutions. Representation of real life problems should begin at the concrete, pictorial level, and eventually at the abstract level.

To ensure students have the prerequisites to automaticity in basic math facts, they need to demonstrate the ability to perform the following:

1. *Count rationally past 100*
2. *Explain that the sequence of counting does not change*
3. *Count objects in any order*
4. *Identify that the number named last in counting refers t o the number of total objects in the set,*
5. *Count past difficult numbers (such as, 19 to 20, 29 to 30, or 100 to 101)*
6. *Count backwards, starting with any number*
7. *Skip count by 2's, 5's, and 10's*
8. *Relate basic addition and subtraction facts*
9. *Explain the operation of multiplication*
10. *Explain place value through the hundreds.*

Once students have mastered the skills listed above and the teacher has provided the motivation to learn number facts through the use of "real life" problems, formal instruction in the multiplication facts can begin. Using the multiplication matrix in Table 1, students can visualize the 100 number facts to be learned. Each student should have a copy of the multiplication grid in a notebook for use during direct instruction. The teacher needs to explain that numbers across the top row are multiplied by the numbers in the first column on the left-hand side. For example, to find the product of 6 x 9, locate the 6 across the top row and the 9 in the first column on the left. Find the number (54) that intersects the 6 and the 9.

Table I

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

with easy rules or those most related to the student's current knowledge. The following are specific strategies addressed in this paper:

1. *Multiplication Matrix*
   - a. *Zero Generalization (N x 0 = 0)*
   - b. *Identity Element (N x 1 = N)*
   - c. *Math 15-Step counting*
   - d. *Commutative Property*
   - e. *Doubles*
2. *NumberPatterns/Personalized Stories*
3. *Finger Math*

1. *Multiplication Matrix*

a. *Zero Generalization*: *The product of any number times zero is always zero*. Have students write out the number facts for zero (e.g., 1x 0=0 through 10x 0=0). Practice with flash cards and always verbalize the property of zero. Once students have mastered this property, have them color in the column and row of "zeros" to demonstrate facts already learned (Figure I).

B. *Identity Elements:* Any number multiplied by 1 will equal that number, N x 1 = N.

Again, have students write out the number facts for 1 (1 x 1 = 1 to 1 x 10 = 10). Practice with flash cards and always verbalize the identity element. Once students have mastered this property ask them to color in the row and column for 1's as shown in Table II.

Table II

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

students to understand skip counting by 2's, place a number line on the floor. Starting with their feet on 0, have them skip l and land on 2, continuing to skip (the odd numbers) up to number 20. (This can also be done with a number line on their desk). First, have students *verbally* rehearse saying their 2's (2, 4, 6, 8, 10, etc.), followed by *writing* out their 2's. Make sure you point out that all the products of the 2 multiplication facts are "even" numbers. Once they have mastered step counting by 2, place the multiplication facts in the following format:

2 x 1 = 2	2 x 6 = 12
2 x 2 = 4	2 x 7 = 14
2 x 3 = 6	2 x 8 = 16
2 x 4 = 8	2 x 9 = 18
2 x 5 = 10	2 x 10= 20

For practice learning any set of multiplication facts, select a student to stand up in front of class and *say* their number facts as they read them off the board. After saying them once, remove the first number fact and have the student read them again including the number erased. This continues until all the facts are erased and the student is saying them from memory. Here is an example of the 2's:

2	4	6	8	10	12	14	16	18	20
__	4	6	8	10	12	14	16	18	20
__	__	6	8	10	12	14	16	18	20

Once the students have mastered this set of facts, have them say them backwards in preparation for division. Then color in the 2 column and row, as shown in Table III.

Table III

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Typically, the next step is to introduce the 3's and 4's. However, students need to use the matrix to visualize what they already know. Since their prior knowledge probably includes skip counting by 5's and 10's, review those multiplication facts first.

When reviewing the 5's, point out that each multiplication fact ends either in a 0 or a 5, and they alternate 0, 5, 0, etc. (Use color for the number in the 1's place to help focus their attention on the number change). Mastery of the 5's allows students to color in the 5 row and column in Table IV.

Table IV

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Fives	*Tens*
5 x 0 = 0	10 x 0 = 0
5 x 1 = 5	10 x 1 = 10
5 x 2 = 10	10 x 2 = 20
5 x 3 = 15	10 x 3 = 30
5 x 4 = 20	10 x 4 = 40
5 x 5 = 25	10 x 5 = 50
5 x 6 = 30	10 x 6 = 60
5 x 7 = 35	10 x 7 = 70
5 x 8 = 40	10 x 8 = 80
5 x 9 = 45	10 x 9 = 90
5 x 10= 50	10 x 10=100

Table V

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

When reviewing the 10's, point out that all multiplication facts end in a zero, and beginning with 10 x 1 =10, the numeral in the tens place (10) increases by one ten each time.

After writing out and verbally rehearsing the 10's, color in the column and row for tens (see Table V). By following this suggested skill sequence students see how they have learned 64 of the 100 multiplication facts, leaving only 36 facts to be learned. At this point include numerous references to the order property of multiplication by introducing the commutative property.

d. Commutative Property: The order of the numbers does not change the product (i.e., 5 x 7 = 35 and 7 x 5 = 35) needs to be reviewed with many other examples. Students should be prompted to look at examples like 7 x 5 and ask themselves what they already know. Understanding the commutative property will enable students to figure out that 7 x 5 = 35 because they already know 5 x 7 = 35. Table VI includes the commutative property by omitting 24 facts (darkened numbers) because they are duplicates of the remaining multiplication tables to be learned.

e. *Doubles:* (8 x 8 and 6 x 6) are included in Table VI and can be isolated and taught through mneumonics, rhymes, stories, etc. For example:

8 x 8 fell on the floor. When they got up they were 64;

4 x 4 were never seen until they started driving at 16;

Learning the doubles will omit four more facts on the matrix, leaving only 15 facts to be learned. Table VI reflects zero generalization, the identity element, the commutative property, the doubles, and the easy facts (i.e., 2's, 5's, & 10's).

Table VI

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

2. *Number patterns:* These will be helpful for learning the more difficult multiplication tables of 3's, 4's, 6's, 7's, 8's, and 9's. The 3's and 7's are probably the most difficult to learn. Identifying the odd/even pattern in the 3's and 7's may help focus student attention to number relationships. Personalizing these numbers through their own experiences may enhance their storage and retrieval of these number facts. The following is an example of a story for the 3's multiplication facts written by a 10-year-old girl with a specific learning disability:

3 x 0 = 0 She was not even born yet

3 x 1 = 3 She went to preschool

3 x 2 = 6 She entered kindergarten

3 x 3 = 9 Last year she would be in single digits

3 x 4 = 12 Last year before she becomes a teenager

3 x 5 = 15 She could get her learner's permit to drive

3 x 6 = 18 She could go away to college and vote

3 x 7 = 21 She would be legal

3 x 8 = 24 She would finally by permitted to date

3 x 9 = 27 This is her birthdate

3 x 10=30 She is allowed to get married

Remind students to use what they already know when they approach a difficult multiplication combination.

For example:

$$\begin{array}{r} 7 \\ \times\ \underline{8} \\ ? \end{array} \qquad \begin{array}{r} 7 \\ \times\ \underline{4} \\ 28 \end{array} \quad = \quad \begin{array}{r} 28 \\ +\underline{28} \\ 56 \end{array}$$

Some students may not know what 7 x 8 equals, but they do know that 7 x 4 = 28, and doubling 28 will give you the product of 7 x 8. Show them other examples, like the one below, where students can use their prior knowledge to figure out an unknown product.

For example:

$$\begin{array}{r} 8 \\ \underline{x7} \\ ? \end{array} \qquad \begin{array}{r} 8 \\ \underline{x5} \\ 40 \end{array} \text{ and } \begin{array}{r} 8 \\ \underline{x2} \\ 16 \end{array} \quad = \quad \begin{array}{r} 40 \\ +\underline{16} \\ 56 \end{array}$$

They may not remember 8 x 7 but they know 8 x 5 and 8 x 2 added together will give them the product for 8 x 7.

The 4's multiplication tables are the two's doubled, and the 8's are the 4's doubled, and both are even numbers with a pattern.

Four's		*Eight's*	
4 x 0 = *0*		8 x 0 = 0	
4 x 1 = 4		8 x 1 = 8	
4 x 2 = 8		8 x 2 = 16	
4 x 3 = 12	There is a pattern: 4	8 x 3 = 24	There is a pattern: 0
4 x 4 = 16	8	8 x 4 = 32	8
4 x 5 = 20	2	8 x 5 = 40	6
4 x 6 = 24	6	8 x 6 = 48	4
4 x 7 = 28	0	8 x 7 = 56	2
4 x 8 = 32		8 x 8 = 64	
4 x 9 = 36		8 x 9 = 72	
4 x 10=40		8 x 10=80	

The 6's multiplication tables are the three's doubled. There is a repeated pattern in the one's place.

6 x 0 = 0	
6 x 1 = 6	
6 x 2 = 12	
6 x 3 = 18	
6 x 4 = 24	There is a pattern: 0
6 x 5 = 30	6
6 x 6 = 36	2
6 x 7 = 42	8
6 x 8 = 48	4
6 x 9 = 54	
6 x 10 =60	

The 9's multiplication tables have a noticeable pattern. When each numeral of a 9 product are added together they equal 9. For example, 9 x 6 = 54 (5 + 4 = 9), and 9 x 8 = 72 (7 2 = 9) Also, starting at the top, the numerals in the one's place start with 9 and go backwards to 0 (9, 8, 7, 6, 5, 4, 3, 2, 1, 0). Starting at the bottom and going up in the tens column, the numerals start with 9 and go to 0.

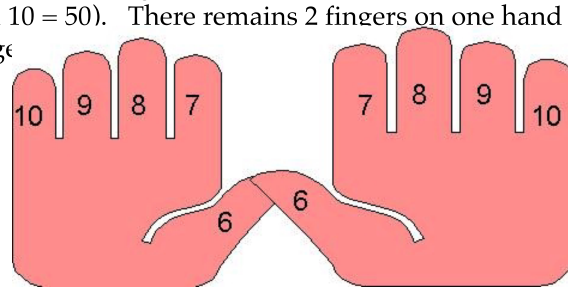
9 x 0 = 0	
9 x 1 = 9	(0 + 9 = 9)
9 x 2 = 18	(1 + 8 = 9)
9 x 3 = 27	(2 + 7 = 9)
9 x 4 = 36	(3 + 6 = 9)
9 x 5 = 45	(4 + 5 = 9)
9 x 6 = 54	(5 + 4 = 9)
9 x 7 = 63	(6 + 3 = 9)
9 x 8 = 72	(7 + 2 = 9)
9 x 9 = 81	(8 + 1 = 9)
9 x 10=90	(9 + 0 = 9)

Finger Math: Some students may still need something concrete to assist them in finding the number facts. *Finger Multiplication* may provide these students with a sense of security.

Finger Multiplication

Place the back of your hands toward you.	Label your thumbs 6, Index finger 7, middle finger 8, ring finger 9, and pinkies 10.	Every finger touching and below are 10's.	Multiply remaining fingers on one hand with the other.	Add numbers together.
1.	2.	3.	4.	5.

Example: 8 x 7. The middle finger on one hand is touching the index finger of the other hand. There are two fingers touching and three below, for a total of 5 fingers (5 x 10 = 50). There remains 2 fingers on one hand and 3 fing



In conclusion, the suggestions outlined in this paper provide educators with strategies to organize the multiplication tables into a format (multiplication grid) to promote rules and number patterns and perhaps offset the traditional drill and memorization approach most often used. The multiplication matrix provides students with a visual representation of the number facts they are learning and a positive consequence for their efforts as they color in these facts they have mastered.

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